Analytical description of directivity patterns of laser ultrasound in diamond anvil cell

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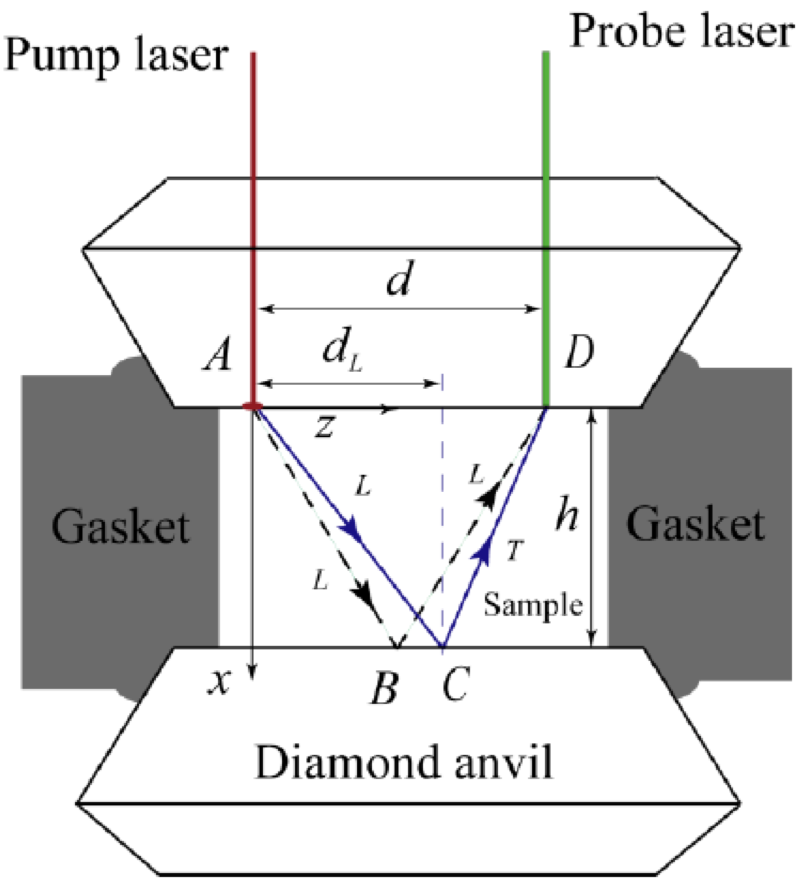
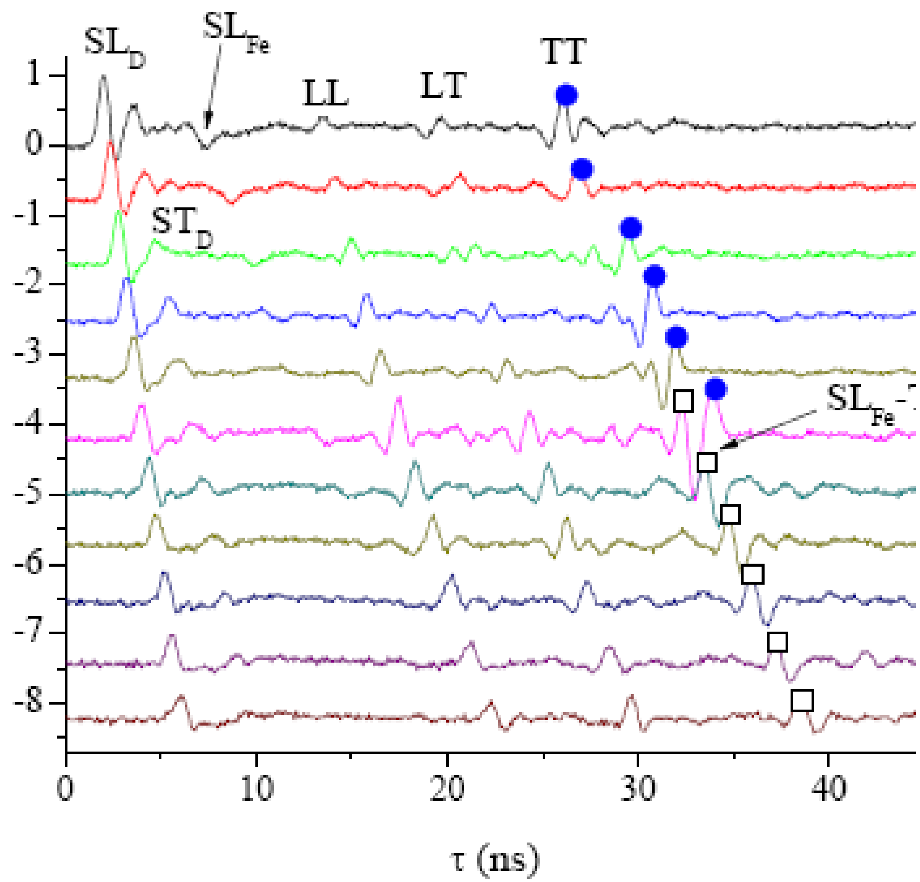
Plan of presentation

1. Concept of LU-DAC technique and motivation
2. Formulation of the problem and the solution
3. Testing of the analytical solution
4. Directivity pattern for Fe in the DAC
5. Conclusions and future work

Concept of LU-DAC technique and Motivation

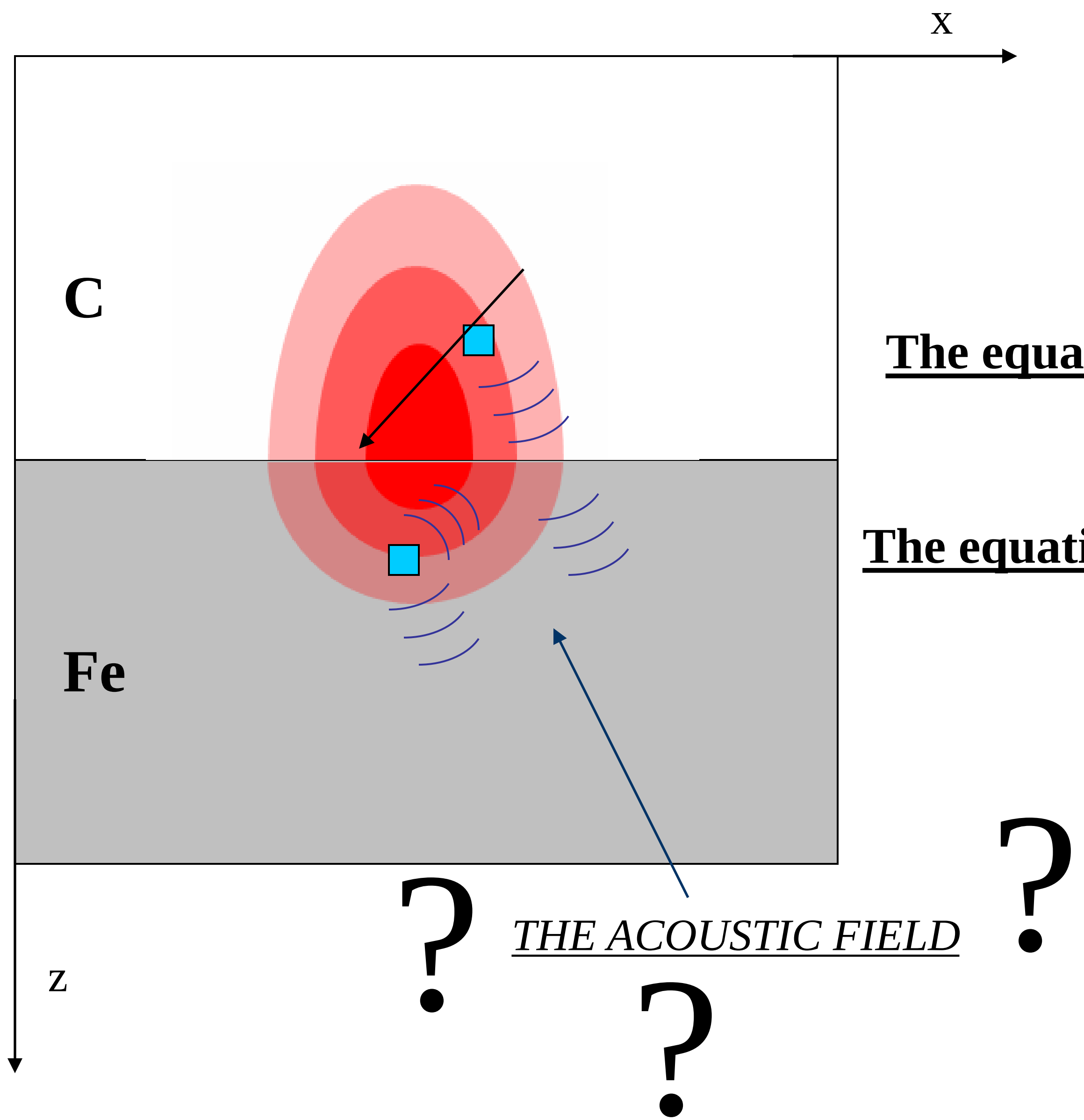


The signals measured at different distances *d*.



the directional pattern → **optimize LU-DAC technique**

Physical model



2D Cartesian coordinate system

The equations for the acoustic field

The equations for the temperature field

Mathematical description of the model

The equations for the acoustic field $\vec{u} = \vec{u}_l + \vec{u}_t \quad \vec{u}_l = grad\phi \quad \vec{u}_t = rot\psi$

x

C(2)

$$\frac{1}{\upsilon_l^2}\frac{\partial^2 \phi_2}{\partial t^2} - \Delta\phi_2 = \sigma_n^2$$

$$\frac{1}{\upsilon_t^2}\frac{\partial^2 \psi_2}{\partial t^2} - \Delta\psi_2 = 0$$

$$\frac{1}{\upsilon_l^2}\frac{\partial^2 \phi_1}{\partial t^2} - \Delta\phi_1 = \sigma_n^1$$

Fe(1)

$$\frac{1}{\upsilon_t^2}\frac{\partial^2 \psi_1}{\partial t^2} - \Delta\psi_1 = 0$$

z

$$\sigma_n = -\alpha\frac{1+\nu}{1-\nu}T$$

$\vec{u}$	acoustic displacement
$\phi \; \psi$	scalar and vector potentials
σ_n	normalized laser-induced stress
$\upsilon_l \; \upsilon_t$	longitudinal and transverse velocity
α	linear thermal expansion coefficient
ν	Poisson ratio
T	temperature

The boundary conditions:

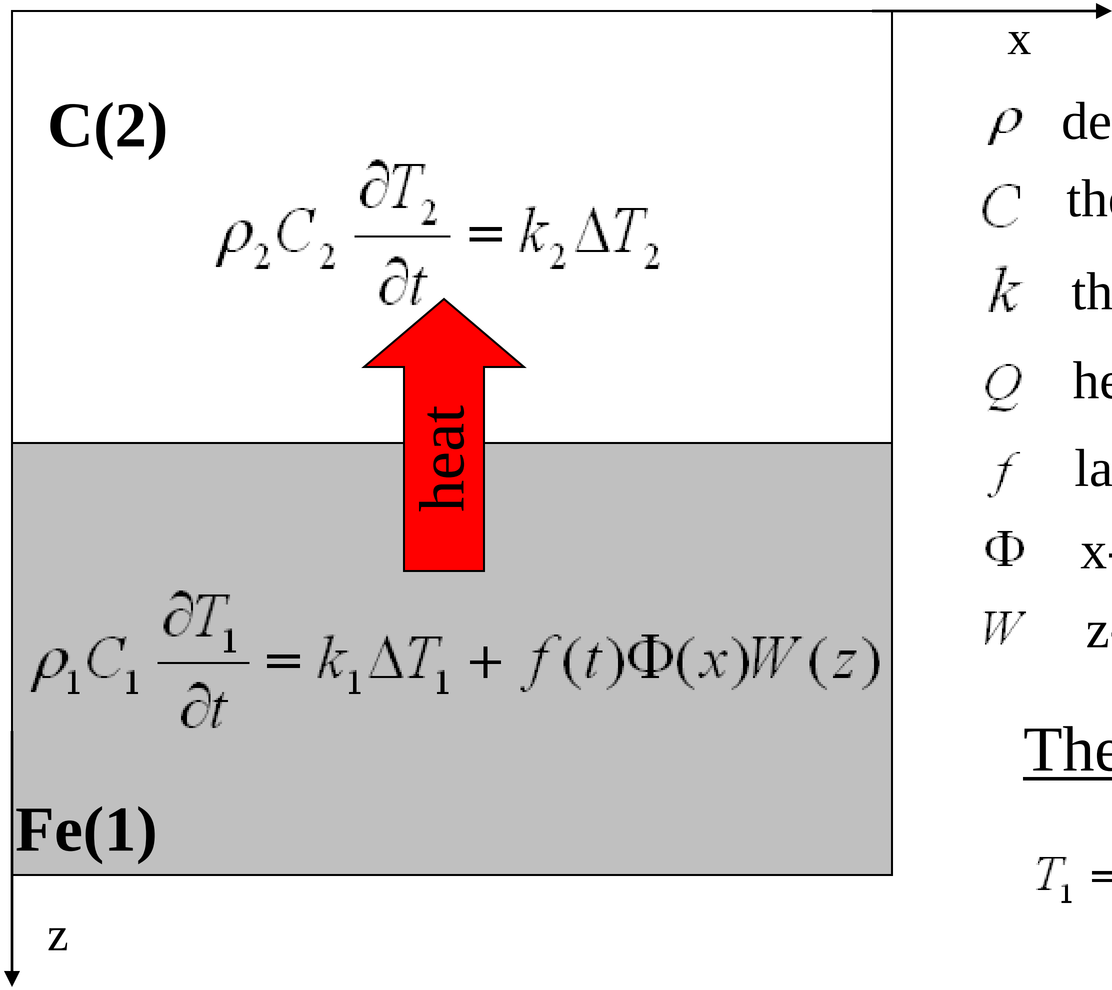
$$u_x^1 = u_x^2 \qquad \sigma_{zz}^1 = \sigma_{zz}^2$$

$$u_z^1 = u_z^2 \qquad \sigma_{zx}^1 = \sigma_{zx}^2$$

Mathematical description of the model

The heat conduction equation:

$$\rho C \frac{\partial T}{\partial t} = k\Delta T + Q$$



$$\rho_2 C_2 \frac{\partial T_2}{\partial t} = k_2 \Delta T_2$$

$$\rho_1 C_1 \frac{\partial T_1}{\partial t} = k_1 \Delta T_1 + f(t)\Phi(x)W(z)$$

- ρ density
- C thermal capacity
- k thermal conductivity
- Q heat source
- f laser pulse intensity envelope in time
- Φ x-distributions of the pulsed laser
- W z-distributions of the pulsed laser

The boundary conditions:

$$T_1 = T_2 \qquad k_1 \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z}$$

Fourier transformation

$$\tilde{\tilde{\tilde{F}}}(\omega, k_x, k_z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(t, x, z) e^{-i(\omega t - k_x x - k_z z)} dt dx dz,$$

$$F(t, x, z) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\tilde{\tilde{F}}}(\omega, k_x, k_z) e^{i(\omega t - k_x x - k_z z)} d\omega dk_x dk_z$$

The basic steps for solving

1) solve the acoustic equations in the Fourier space

2) integrate over the K_z (the theory of residues)

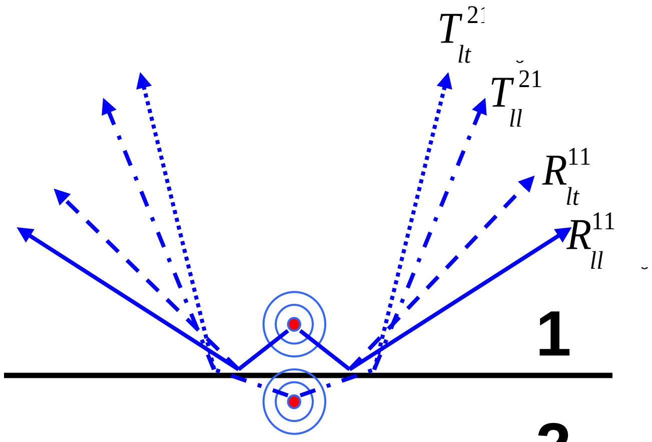
3) the use of boundary conditions (Stoneley determinant)

4) integrate over the K_x (method of the stationary phase)

5) transition into polar coordinates

+ solve the heat conduction equation in the Fourier space

Solution



$$\omega / \upsilon \equiv k$$

$$\begin{pmatrix} \beta_1 & \beta_2 & -k_x & k_x \\ k_x & -k_x & \alpha_1 & \alpha_2 \\ \beta_1 & \mu_{21}\beta_2 & -\gamma_1 & \mu_{21}\gamma_2 \\ \gamma_1 & -\mu_{21}\gamma_2 & \alpha_1 & \mu_{21}\alpha_2 \end{pmatrix} \begin{pmatrix} R_{lt}^{11} \\ T_{lt}^{12} \\ R_{ll}^{11} \\ T_{ll}^{12} \end{pmatrix} = \begin{pmatrix} k_x \\ \alpha_1 \\ \gamma_1 \\ \alpha_1 \end{pmatrix}$$

The reflection and transmission coefficients

$$k_x = \begin{cases} k_{l1} \sin \theta, \leftrightarrow \tilde{\phi}_1 \\ k_{t1} \sin \theta, \leftrightarrow \tilde{\psi}_1 \end{cases}$$

$$\gamma_{1,2} \equiv k_x - k_{t1,2}^2 / (2k_x)$$

$$\beta_{1,2} \equiv \sqrt{k_{t1,2}^2 - k_x^2}$$

$$\alpha_{1,2} \equiv \sqrt{k_{l1,2}^2 - k_x^2}$$

The approximations:

1) two-dimensional space
2) far-field region
3) sub-nanosecond laser

Solution

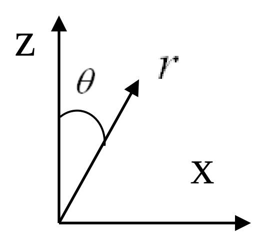
$$\tilde{\phi}_1(\omega, r, \theta) = P_1 I \left\{ 1 + R_{ll}^{11}(\theta) + P_{2/1} \frac{\cos\theta}{\sqrt{(\upsilon_{l1}/\upsilon_{l2})^2 - \sin^2\theta}} T_{ll}^{21}(\theta) \right\} \frac{\tilde{\Phi}(k_{l1}\sin\theta)\tilde{\mathcal{F}}(\omega)}{\omega} \sqrt{\frac{i}{2\pi k_{l1} r}} e^{-ik_{l1}r}$$

$$\tilde{\psi}_1(\omega, r, \theta) = P_1 I \left\{ \frac{\cos\theta}{\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2 - \sin^2\theta}} \left[R_{lt}^{11}(\theta) + P_{2/1} \frac{\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2 - \sin^2\theta}}{\sqrt{(\upsilon_{t1}/\upsilon_{l2})^2 - \sin^2\theta}} T_{lt}^{21}(\theta) \right] \right\} \frac{\tilde{\Phi}(k_{t1}\sin\theta)\tilde{\mathcal{F}}(\omega)}{\omega} \sqrt{\frac{i}{2\pi k_{t1} r}} e^{-ik_{t1}r}$$

$$\left|\Xi_\phi(\theta)\right| \equiv \left| 1 + R_{ll}^{11}(\theta) + P_{2/1} \frac{\cos\theta}{\sqrt{(\upsilon_{l1}/\upsilon_{l2})^2 - \sin^2\theta}} T_{ll}^{21}(\theta) \right|$$

$$\left|\Xi_\psi(\theta)\right| \equiv \left| \frac{\cos\theta}{\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2 - \sin^2\theta}} \left[R_{lt}^{11}(\theta) + P_{2/1} \frac{\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2 - \sin^2\theta}}{\sqrt{(\upsilon_{t1}/\upsilon_{l2})^2 - \sin^2\theta}} T_{lt}^{21}(\theta) \right] \right|.$$

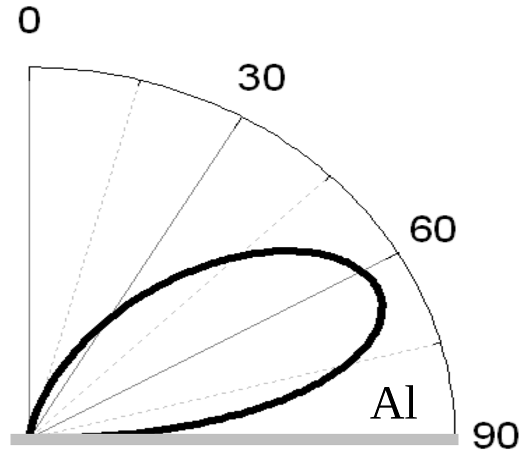
$$P_{2/1} \equiv \frac{\alpha_2(1+\nu_2)(1-\nu_1)\sqrt{\chi_2}}{\alpha_1(1+\nu_1)(1-\nu_2)\sqrt{\chi_1}} \qquad \chi = \frac{k}{\rho C}$$

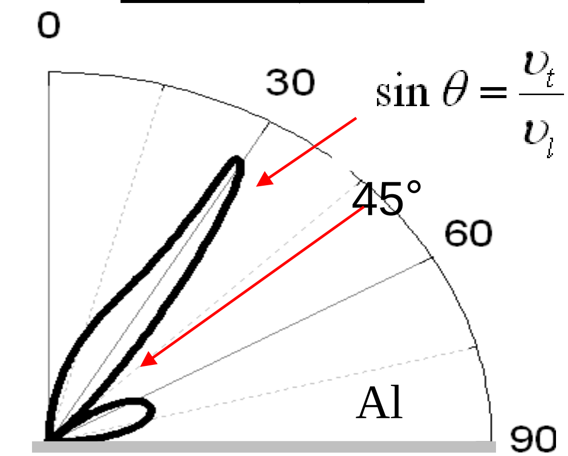


Directivity pattern for the free surface

Longitudinal(Al) : | **Shear(Al) :**

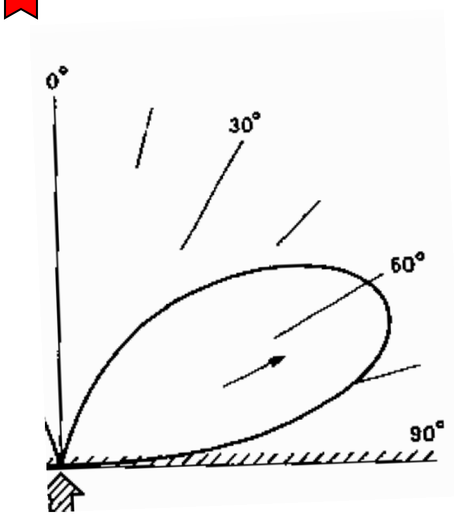
Our solution

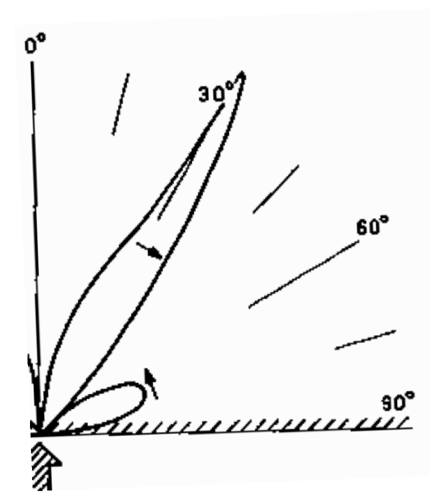




$$\sin\theta = \frac{\upsilon_t}{\upsilon_l}$$

Early work[1]

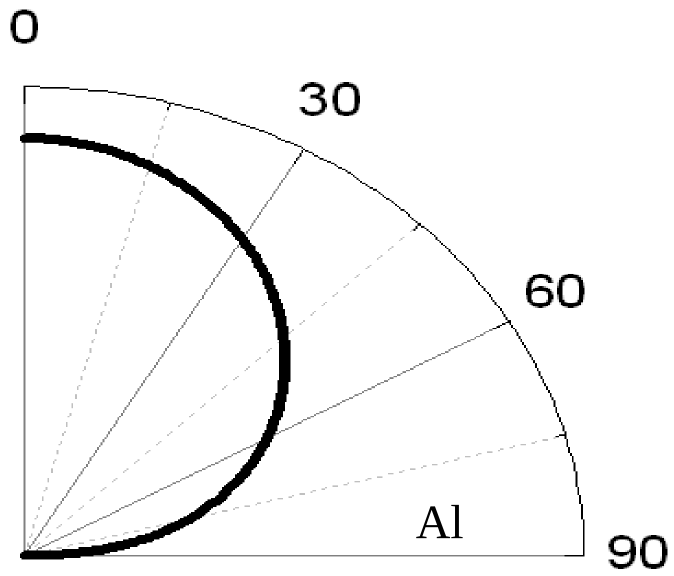




[1]C.B. Scruby, L.E. Drain, “Laser ultrasonics. Techniques and applications”, NY, Adam Hilger (1990).

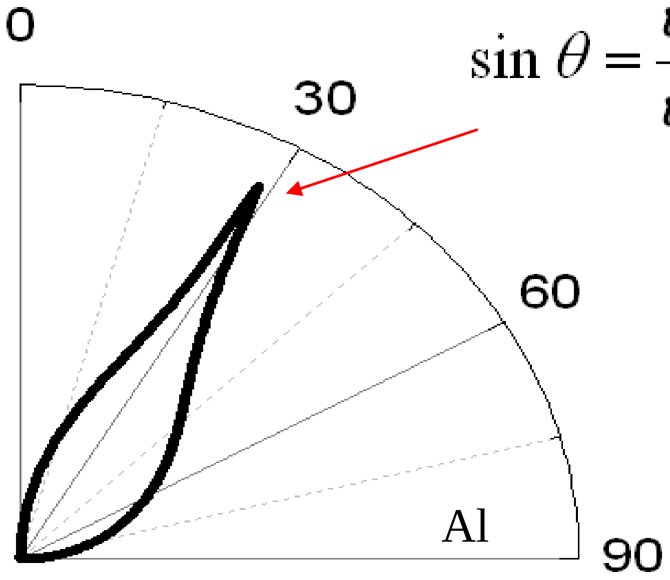
Directivity pattern for the rigid surface

Longitudinal(Al) :



Shear(Al) :

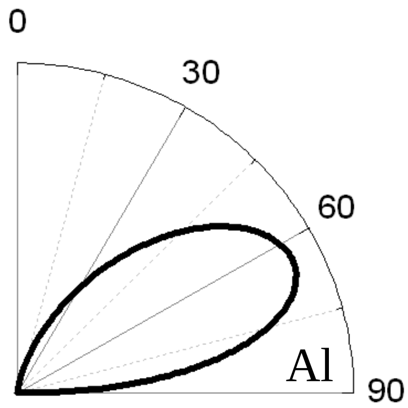
$$\sin\theta = \frac{\upsilon_t}{\upsilon_l}$$

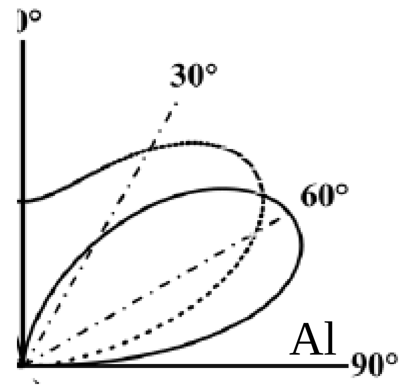


Comparison with earlier works

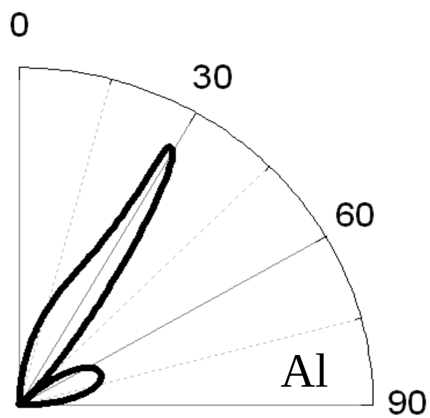
Our solution | **Wen Feng et al.**

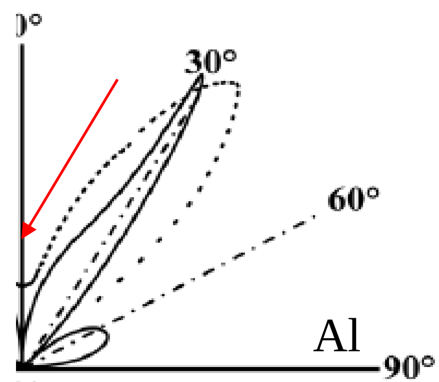
aluminium for free surface, Longitudinal





aluminium for free surface, Shear





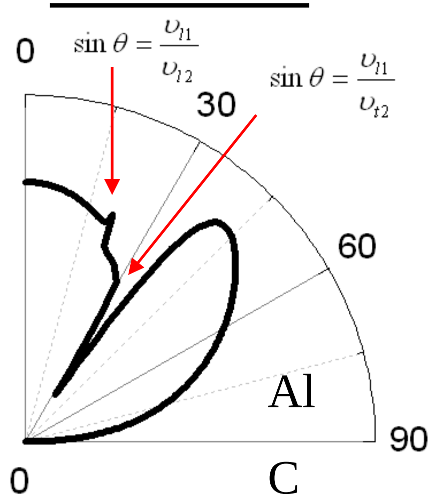
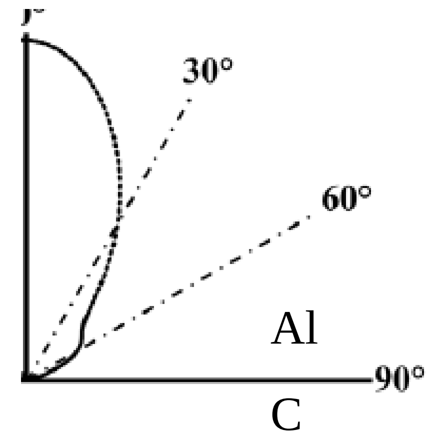
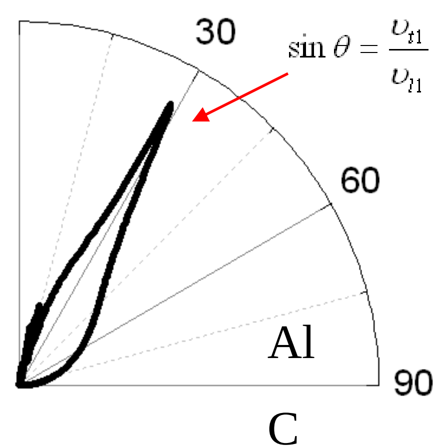
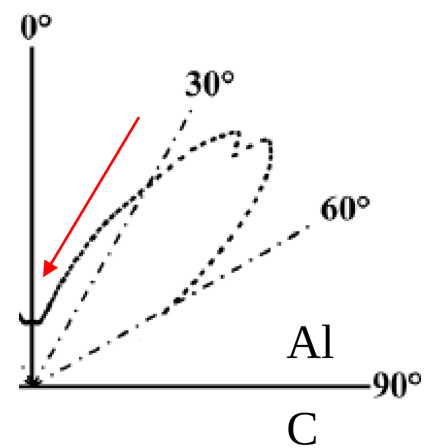
——— **Dipol model**

- - - - - - - **FEM**

point source
single frequency

(distributed source
broadband pulse)

Comparison with earlier works

	Our solution	**Wen Feng et al.**
aluminium in DAC at 0 GPa, Longitudinal		
aluminium in DAC at 0 GPa, Shear		

point source
single frequency

FEM
(distributed source
broadband pulse)

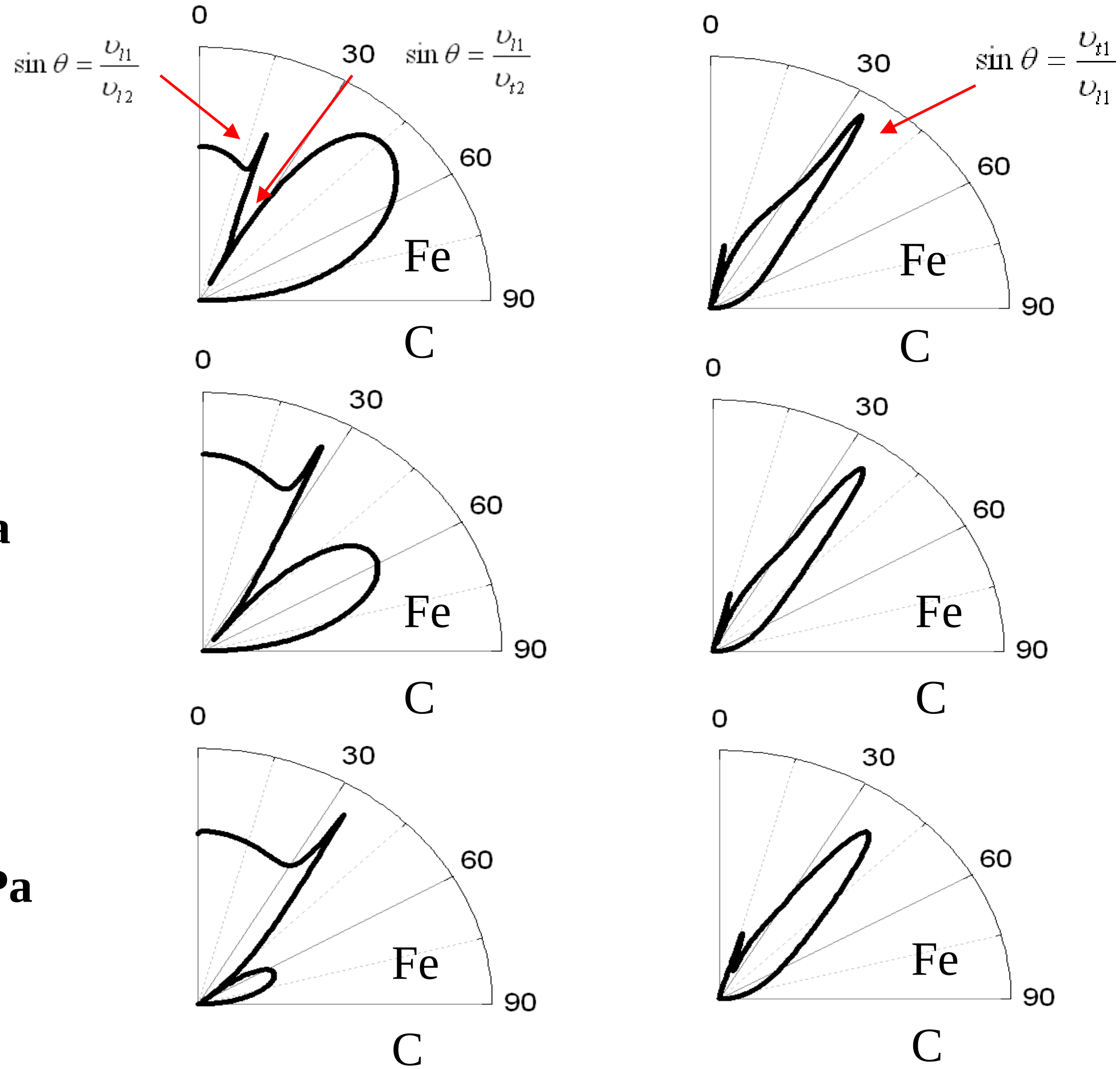
Directivity pattern for Fe in the DAC

Longitudinal | **Shear**

Iron in DAC at 0 GPa

Iron in DAC at 40 GPa

Iron in DAC at 140 GPa



Future work

1) take into account the size of the source

$$\tilde{\varphi}_1(\omega, r, \theta) = P_1 I \left\{ 1 + R_{ll}^{11}(\theta) + P_{2l1} \frac{\cos\theta}{\sqrt{(\upsilon_{l1}/\upsilon_{l2})^2 - \sin^2\theta}} T_{ll}^{21}(\theta) \right\} \frac{\tilde{\Phi}(k_{l1}\sin\theta)\tilde{f}(\omega)}{\omega} \sqrt{\frac{i}{2\pi k_{l1} r}} e^{-ik_{l1} r}$$

$$\tilde{\psi}_1(\omega, r, \theta) = P_1 I \left\{ \frac{\cos\theta}{\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2 - \sin^2\theta}} \left[R_{lt}^{11}(\theta) + P_{2l1} \frac{\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2 - \sin^2\theta}}{\sqrt{(\upsilon_{t1}/\upsilon_{l2})^2 - \sin^2\theta}} T_{lt}^{21}(\theta) \right] \right\} \frac{\tilde{\Phi}(k_{t1}\sin\theta)\tilde{f}(\omega)}{\omega} \sqrt{\frac{i}{2\pi k_{t1} r}} e^{-ik_{t1} r}$$

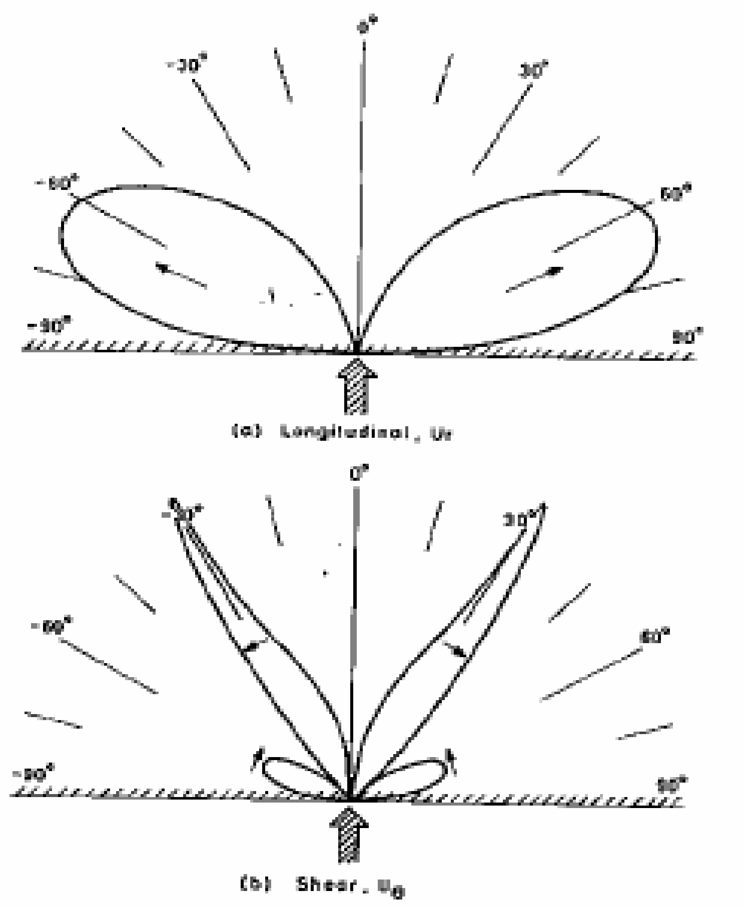
2) comparison with experiment

3) calculation profiles of acoustic pulses

Conclusions

- Analytical description of directivity patterns of laser ultrasound in diamond anvil cell was obtained
- Tests of the solutions were conducted
- Directivity pattern for Fe in the DAC was obtained
- The results could be used for the optimisation of future laser ultrasonic experimental set-up in DAC

Thank you for your attention



C.B. Scruby, L.E. Drain, “Laser ultrasonics. Techniques and applications”, NY, Adam Hilger (1990).

$$\left[\frac{2\cos\theta\sqrt{(\upsilon_{l1}/\upsilon_{t1})^2-\sin^2\theta}}{(\sin\theta-(\upsilon_{l1}/\upsilon_{t1})^2/2\sin\theta)^2+\cos\theta\sqrt{(\upsilon_{l1}/\upsilon_{t1})^2-\sin^2\theta}}\right]$$

$$\left[\frac{2\cos\theta\sqrt{(\upsilon_{l1}/\upsilon_{t1})^2-\sin^2\theta}}{\sin^2\theta+\cos\theta\sqrt{(\upsilon_{l1}/\upsilon_{t1})^2-\sin^2\theta}}\right]$$

$$\left[\frac{2(\sin\theta-1/2\sin\theta)\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2-\sin^2\theta}}{(\sin\theta-1/2\sin\theta)^2+\cos\theta\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2-\sin^2\theta}}\right]$$

$$\left[\frac{2\sin\theta\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2-\sin^2\theta}}{\sin^2\theta+\cos\theta\sqrt{(\upsilon_{t1}/\upsilon_{l1})^2-\sin^2\theta}}\right]$$